COOLING IN THE LOWER ATMOSPHERE AND THE STRUCTURE OF POLAR CONTINENTAL AIR

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It has been known for some time that during a clear, calm night the temperature of a snow surface falls much below the temperature of the air within even a few centimeters of the surface. The snow surface radiates energy practically as a black body; and receives, by conduction from the snow and air, and also by radiation from the atmosphere, less energy than it loses through its own radiation; and consequently its temperature decreases. However, if the wind movement increases, considerable heat is transported to the surface from the warmer air above, by turbulence, and the surface temperature rises. When the wind ceases, the surface temperature again falls. The question arises as to how much the surface temperature can fall without also a simultaneous decrease of the temperature of large portions of the air above it. It will be shown, from considerations of radiative balance between the air and the snow surface, that it is impossible for the surface temperature to fall below a certain value without a simultaneous decrease of the maximum free-air temperature. In other words, it will be shown how, apart from conduction and mechanical turbulence, cooling from below extends upward through the lower troposphere.

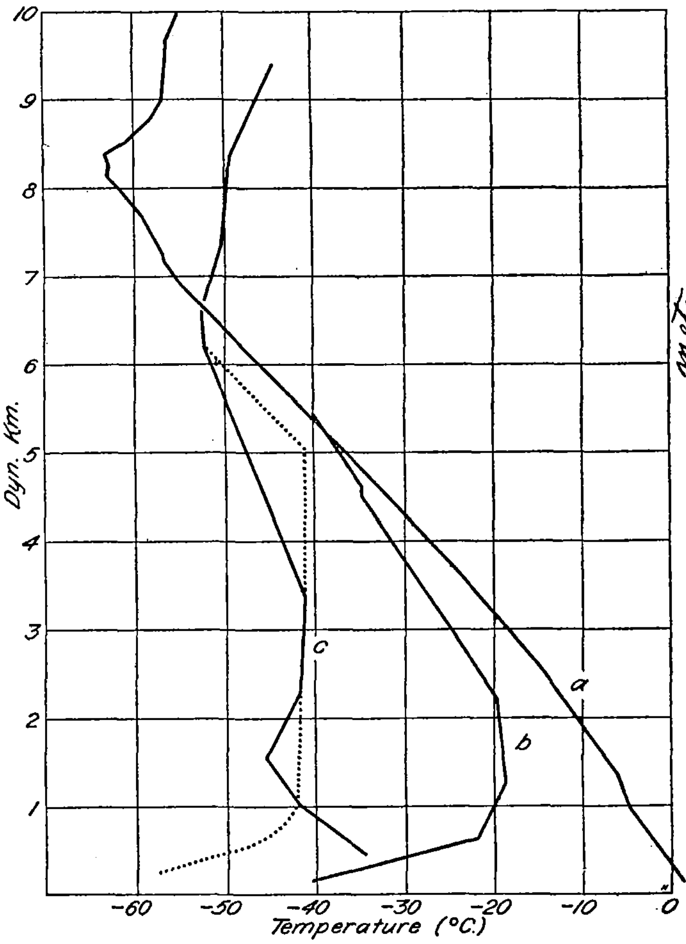

FIGURE 1.—Soundings at (a) Ås, Norway, February 2, 1933, 15.11 h; (b) Fairbanks, Alaska, December 28, 1932, 9.34 h; (c) Ellendale, N. Dak., February 8, 1933, 13.28 h, local time.

THE CHANGE FROM POLAR MARITIME TO POLAR CONTINENTAL AIR MASS CHARACTERISTICS

No better introduction to the problem can be given than by presenting actual soundings showing different types of polar air.

Curve (*a*) in figure 1 shows a portion of a balloon sounding taken through fresh polar maritime air at Ås, Norway (59°40′ N., 10°46′ E.) February 2, 1933. The synoptic situation for this period is discussed by Palmén (1), and it is seen from his maps that the air moved rapidly from the north over the Atlantic Ocean and appeared at Ås with westerly winds. According to Palmén, the air above the surface layer was in almost neutral equilibrium for moist air, and caused numerous showers, hailstorms, and thunderstorms along the Norwegian Coast.

Curve (*b*) is a sounding made at Fairbanks, Alaska (64°51′ N., 147°52′ W.), December 28, 1932, under clear skies and in a calm. An inversion of over 20° occurs between the surface and 1,300 meters, above which is an isothermal layer extending to about 2,200 meters, and a steeper lapse rate above. The wind above the inversion was westerly backing to southwesterly, 15 ~~miles~~ meters per second, at 3,400 meters, showing that the air above the inversion came from the Pacific Ocean. The temperature of this air differs from that of the Ås air by only a few degrees and is practically the same at 5 kilometers. The lower air was shielded from a maritime influence by the extensive mountain ranges that surround the Yukon Valley, and had probably remained over the snow cover for some time.

Curve (*c*) represents a portion of a balloon sounding made at Ellendale, N. Dak. (45°59′N., 98°34′ W.) on February 8, 1933, at about noon in a northwest wind of about 10 meters per second. It shows a superadiabatic surface layer, a smaller adiabatic layer, then an inversion, and isothermalcy to about 3,300 meters, above which the lapse rate steepens. The synoptic situation for this period is discussed by Willett (2), and it shows that the sounding was taken in a well developed current of polar continental air from the Hudson Bay area. The steep lapse rate in the lower air is a result of mechanical turbulence and heating from below. Before the air started its rapid movement southward it probably possessed a thermal structure in its lower portion similar to that indicated by the dotted line, as will be shown presently.

The problem that confronts us is to explain how sounding (*a*) is transformed into sounding (*c*). That such a transformation takes place over the polar regions during winter can hardly be doubted, for the constant drain of polar continental air from its source region southward must be compensated by an inflow of warmer air. The problem is in no way restricted if we assume as an initial air mass a body of polar maritime air, such as represented by the Ås sounding. In order to transform the Ås sounding into the Ellendale sounding, it is evident that we must seek an explanation that involves radiative transfer; and since the maximum temperature difference occurs at lower levels (38° C. at 1,500 meters) and the two soundings approach each other aloft (5° C. difference at 6,000 meters), it appears that heat is lost to space mostly by way of the snow surface; we shall therefore examine how this can take place.

According to Falckenberg (3) the amount of radiation emitted by a snow surface (frozen or melting) differs by less than one-half percent from that coming from a black body, and so follows closely the law of Stefan-Boltzmann,

$$I=\sigma T^4,$$

where I is the radiation intensity in gm. cal./cm²/min., $\sigma=82\times10^{-12}$, T=temperature in degrees absolute.

The radiative properties of the atmosphere are much more complex, since the air is composed of several gases, only a few of which are good absorbers of long wave radiation. In the wave-length region for which terrestrial radiation is most intense, carbon dioxide has an important absorption band; but the best absorber is water vapor, and its varying content in the atmosphere combined with the complicated fashion in which its absorption coefficient depends on wave-length, adds to the difficulty of treating the problem analytically. Indeed, judging from the disagreement among various investigators, the exact nature of the water vapor absorption spectrum is still unknown. Some of the chief investigators have been Rubens and Aschkinass (4), Fowle (5), Hettner (6), and, more recently, Weber and Randall (7). Each of these authorities agrees to the existence of a strong, narrow, absorption band at about 7μ, a region of low absorption centered at about 10μ, and increasing absorption beyond 12μ. However, their quantitative results are not in agreement; Hettner's absorption values are the largest, while those of Weber and Randall are the least; in between these two extremes are the determinations of Rubens and Aschkinass and of Fowle.

Hettner's data have usually been used by most meteorologists, especially by Simpson (8), who applied them with considerable success to certain problems concerning the mean heat balance of the atmosphere. Recently, however, Ramanathan and Ramdas (9), in trying to derive theoretically Ångström's formula for atmospheric radiation, found that Hettner's absorption data led to a value of a constant in the formula which was much larger than the observed one, while the data of Weber and Randall gave a value in much better agreement.

In figures 2a and 2b are shown the laboratory results of Hettner and of Weber and Randall. Against wave-length is plotted α, the decimal coefficient of absorption, which, assuming Beer's law to be true for water vapor, is defined in the following manner:

$$10^{-\alpha m}=I_\lambda/I_{0\lambda},$$

where $I_{0\lambda}$ is the intensity of the incident radiation of wave-length, λ, and
I_λ is the intensity after the radiation has passed through m gms. of the absorbing material.

The two curves in figure 2a, and Hettner's curve in figure 2b have been reproduced from the paper of Ramanathan and Ramdas; but the Weber-Randall curve in figure 2b has been prepared by the writer, from the data published by Weber and Randall, together with certain additional information kindly supplied by Professor Randall. Ramanathan and Ramdas evidently assumed the temperature of the absorption tube that contained the saturated water-vapor to be about 30° C., whereas it was actually about 22.5° C. This means that the actual absorption coefficients are higher than those calculated by Ramanthan and Ramdas. This correction has been applied to figure 2b but not to figure 2a, since it is negligible for low values of absorption. Also, due to an error in printing, 10 absorption lines, 21.39μ to 22.55μ, inclusive, were included in the steam absorption data instead of the water vapor data; this accounts for the apparent low absorption shown in this region in the paper of Ramanathan and Ramdas.

The curves in figures 2a and 2b show that Weber and Randall found smaller absorption than did Hettner. The former authorities used a 60° potassium bromide prism twice, resulting in high dispersion and resolution, while Hettner used a 20° potassium chloride prism and larger slit widths. Professor Randall has assured the writer that "it is quite impossible to compare results when such a difference exists." In this study, both sets of absorption data are used, and their respective meteorological consequences examined and compared.

Simpson's use of the absorption data of Hettner (for water vapor) and Rubens and Aschkinass (for carbon dioxide).—We shall assume with Simpson (8) that a layer of air in which each column of unit cross section contains 0.15 millimeters precipitable water and 0.03 gram of CO_2[1] completely absorbs all radiation of wave-lengths between 5.5μ and 7μ, and greater than 14μ; partially absorbs radiation of wave-lengths between 4μ and 5.5μ, 7μ and 8.5μ, 11μ and 14μ; and is practically transparent to wave-lengths between 8.5μ and 11μ. These wave-lengths together cover the range over which terrestrial and atmospheric radiation have appreciable intensity. To avoid repetition, let us call the radiation with wave-lengths in the first group, *W*-radiation (after Brunt); in the second, *S*-radiation; and in the last, *T*-radiation. At a temperature of 273°, the percentages of radiation from a black body included in these three groups are 59%, 26%, 15%, respectively.

The length of a column of air of 1 square centimeter cross section which contains 0.15 millimeter precipitable water is given approximately by

$$L=0.70\frac{T}{e}\text{meters},$$

where T is the mean temperature of the column in °A, and e is the mean vapor pressure in mb; when $T=273°$ and $e=6$ $mb.$, then $L=40$ meters, and when $T=253°$ and $e=1$ $mb.$, then $L=175$ meters.

If we divide the atmosphere into layers each of whose unit columns contains 0.15 millimeter precipitable water, then the amount of CO_2 in each unit column of the layer ordinarily will not be of the order of 0.03 gram. According to Fowle (10), the average sea-level content of CO_2 is 0.6 gram per cubic meter. Assuming this value is the same above the surface (probably it is too high), then a column of air of cross-section 1 square centimeter, and 40 meters long, will contain only 0.0024 gram of CO_2; and a column 200 meters thick, 0.012 gram. Now as found by Rubens and Aschkinass (4), CO_2 absorbs very strongly in a narrow band from 13 to 16μ, with its maximum absorption occurring at 14.7μ and equal to 88 percent for 0.06 gram. Assuming 0.06 gram of CO_2 in the stratosphere, Simpson felt justified in decreasing the lower limit of Hettner's complete absorption region, for radiation passing through the stratosphere, from about 20μ to 14μ. If account is taken of the actual CO_2 content in the layers

[1] Because of the diffuse nature of terrestrial and atmospheric radiation, the mean path of the radiation will include approximately twice the amount of absorbing gas; hence, diffuse radiation passing through a layer, each of whose unit columns contains 0.15 millimeter water and 0.03 gram CO_2 will be absorbed in approximately the same proportion as a parallel beam passing through a layer each of whose unit columns contains 0.30 millimeter of H_2O and 0.06 gram CO_2.

considered above, then the absorption will not be so complete as that assumed by Simpson. However, since in this paper the extreme values of absorption will each be used, it will be assumed that in this case the *W*-radiation is contained in the wave-length regions adopted by Simpson.

Now let us suppose that a body of air in convective equilibrium with an open ocean surface of temperature 1° C., and containing no clouds, comes to rest over an unlimited horizontal snow surface during the polar night. Initially, the temperature of the snow surface will be 0° C. and, after stagnation of the air ensues, will fall very rapidly to a value determined by the amount of radiation that is received from the atmosphere (assuming that conduction from the atmosphere and underlying snow is very small). If we divide the atmosphere into layers, each of whose unit columns contains 0.15 millimeter precipitable water, then practically all of the *W*-radiation reaching the surface will originate in the lowest layer. No *T*-radiation will reach the surface, since it is not emitted in any appreciable quantity by the water-vapor. A certain amount of *S*-radiation will reach the surface, and this will come from various layers, the more intense radiation coming from lower layers and the less intense from the higher layers. It is impossible to determine exactly the amount of *S*-radiation that will reach the surface, but it is clear that the radiation in each band must be bounded by a curve extending from full absorption at the temperature of the lowest layer, to complete transparency, the shape of which will depend on the amount and distribution of water vapor. As Simpson pointed out, almost any reasonable curve in these bands, joining full absorption to complete transparency, divides the area approximately in half; hence, we assume the amount of *S*-radiation that reaches the surface to be equal to one-half of the total S-radiation of a black-body at the temperature of the lowest layer.

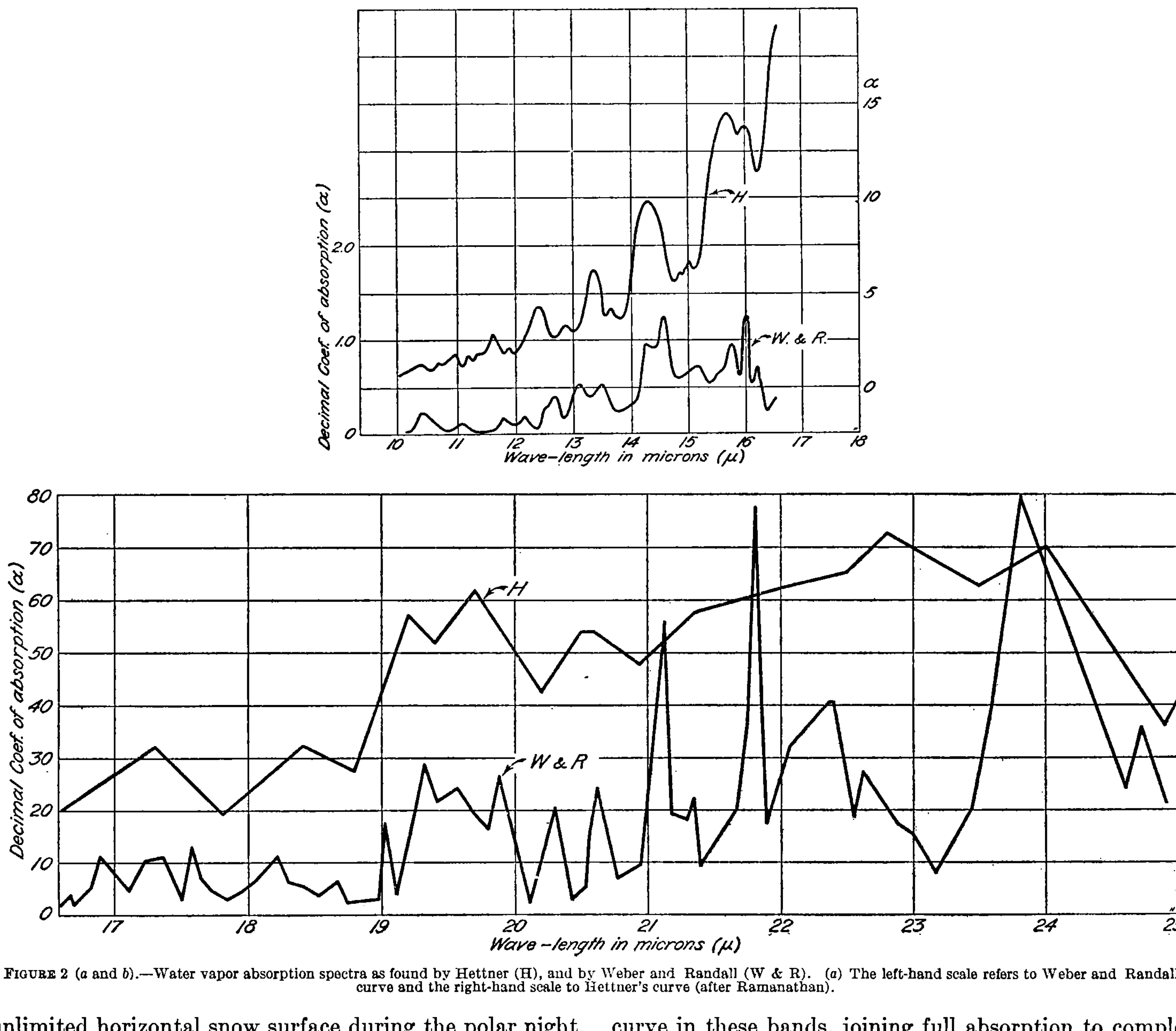

FIGURE 2 (a and b).—Water vapor absorption spectra as found by Hettner (H), and by Weber and Randall (W & R). (a) The left-hand scale refers to Weber and Randall's curve and the right-hand scale to Hettner's curve (after Ramanathan).

In figure 3a, a curve has been drawn that shows the distribution of energy emitted by a black-body at a temperature of 274° A. The amount of radiation that the snow surface receives will be approximately equal to the shaded portion of the area under this curve. Initially, the snow surface, when at a temperature of 273° A., will emit radiation which will be almost equal to the *total* area under this curve; and its temperature will decrease, since it is losing more radiation than it is receiving, the net loss being nearly equal to the unshaded portion of the area. However, at a certain temperature (in this case, 253° A., as will be shown later) the snow surface will be emitting as much radiation as it is receiving from the atmosphere, the temperature of the latter being assumed unchanged. This equilibrium is shown in figure 3a by the equality of the shaded area under the 274° A. curve and the total area under the 253° A. curve. In this case, the surface temperature will remain constant at 253° A. provided that the temperature of the air remains unchanged.

Application of Weber and Randall's absorption data to an atmosphere containing normal amounts of CO_2.—Water vapor absorption as determined by Weber and Randall is so much less than that found by Hettner in the region 10μ–25μ, that in order to estimate the amount of atmospheric radiation in the manner outlined above, it is

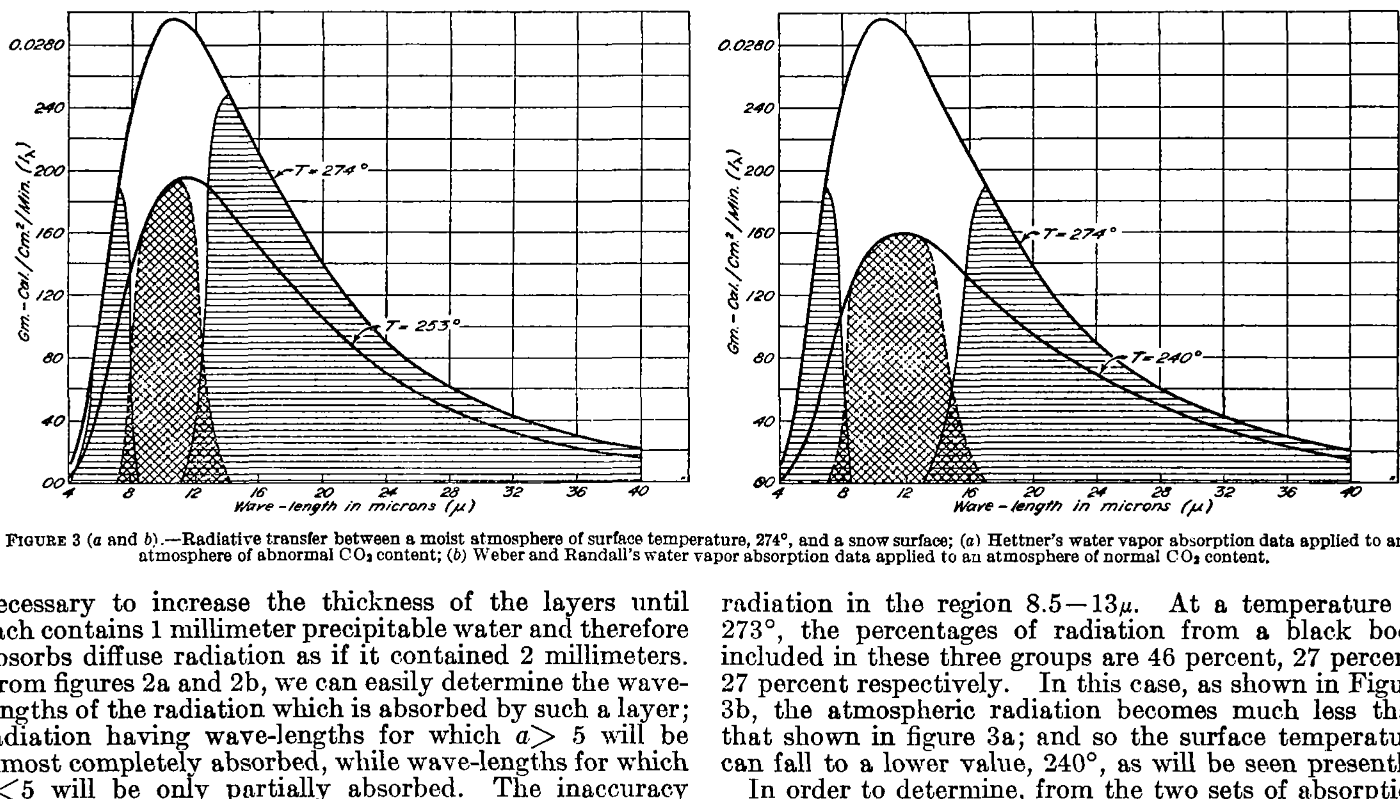

FIGURE 3 (a and b).—Radiative transfer between a moist atmosphere of surface temperature, 274°, and a snow surface; (a) Hettner's water vapor absorption data applied to an atmosphere of abnormal CO_2 content; (b) Weber and Randall's water vapor absorption data applied to an atmosphere of normal CO_2 content.

necessary to increase the thickness of the layers until each contains 1 millimeter precipitable water and therefore absorbs diffuse radiation as if it contained 2 millimeters. From figures 2a and 2b, we can easily determine the wave-lengths of the radiation which is absorbed by such a layer; radiation having wave-lengths for which $a > 5$ will be almost completely absorbed, while wave-lengths for which $a < 5$ will be only partially absorbed. The inaccuracy involved in regarding these thick layers as elementary layers will not be serious except at low temperatures. The length of a unit column containing 1 millimeter precipitable water is

$$L = 4.62 \frac{T}{e} \text{ meters;}$$

when $T = 273°$, and $e = 6mb.$, then $L = 210$ meters; and when $T = 253°$, and $e = 1\ mb.$, then $L = 1{,}170$ meters.

In figures 2a and 2b, we see from Weber and Randall's absorption curve that a is very small from 10μ to 12μ, increases slightly near 13μ, increases markedly near 17μ, and remains quite large beyond 17μ. For the region 10μ to 13μ, the mean value of a is about 0.21, and the transmission of diffuse radiation in this band through a layer containing 1 millimeter precipitable water vapor is more than 90 percent; for the region 13μ–17μ, the mean value of a is about 0.8, corresponding to about 70 percent transmission, while for wave lengths greater than 17μ, the mean transmission is less than 10 percent.

If, now, assuming Fowle's value of 0.6 gm/m^3 as the normal CO_2 content to hold true at all elevations,[2] we also take account of CO_2 absorption in the band 13μ–16μ, then the mean transmission in this band through both the CO_2 and water vapor in a layer 200 millimeters thick is 48 percent; and through a layer 1,000 meters thick, 31 percent. Hence, we shall now define again the W, S, and T bands of radiation to conform with Weber and Randall's absorption data, and Fowle's value of the normal sea-level CO_2 content. The W-radiation will be that contained in the region $5.5-7.0\mu$, and in the region where wave-lengths are larger than 17μ; the S-radiation in the regions $4.0-5.5\mu$, $7.0-8.5\mu$, and 13μ–17μ; and the T radiation in the region $8.5-13\mu$. At a temperature of 273°, the percentages of radiation from a black body included in these three groups are 46 percent, 27 percent, 27 percent respectively. In this case, as shown in Figure 3b, the atmospheric radiation becomes much less than that shown in figure 3a; and so the surface temperature can fall to a lower value, 240°, as will be seen presently.

In order to determine, from the two sets of absorption data, the equilibrium surface temperatures for various air temperatures, three curves have been drawn in figure 4, one showing the variation of black-body radiation with temperature, the second showing the variation of atmospheric radiation with temperature, based on Simpson's application of Hettner's data in an atmosphere considerably overcharged with CO_2, and the third showing the same as the second but making use of Weber and Randall's data in an atmosphere containing more nearly a normal amount of CO_2. For comparison, in figure 4, are plotted the mean values of atmospheric radiation against surface temperature as observed on the "Maud" north of the Siberian coast by Sverdrup and discussed by Mosby (11). These are indicated by crosses, and were found by Mosby

[2] This value is probably too large for the free air, and also for surface winter polar conditions where on account of lack of animal and industrial activity the CO_2 content is probably very small.

by subtracting the mean nocturnal radiation from the black body radiation corresponding to the surface temperature. The individual observations, when plotted against surface temperature, exhibit considerable scatter, probably indicating that the radiation was not so much a function of surface temperature as of the air temperature above the inversion; unfortunately, due to lack of sufficient soundings at the time of the radiation measurements, it was not possible to show this. In order to compare the observations with curves (*b*) and (*c*) which represent atmospheric radiation plotted against temperature of the air above the inversion, the crosses must be moved a certain number of degrees to the right, corresponding to the mean value of the surface inversion.

presence of larger inversions over the polar regions than over the temperate regions.

When curves (*b*) and (*c*) are plotted on logarithmic paper, they are approximately straight lines; and their equations are found to be

$$(b)\ R_A=980\times10^{-12}\times T^{3.5}\ \text{gm cal/cm}^2\text{/min;}$$
$$(c)\ R_A=800\times10^{-12}\times T^{3.5}\ \text{gm cal/cm}^2\text{/min.}$$

The value 3.5 for the exponent was also found by Pekeris (**13**), from the data of Abbot and Fowle (**14**), (**5**). Pekeris plotted logarithm of temperature against logarithm of radiation from layers of air containing normal amounts of CO_2 and the following amounts of precipitable

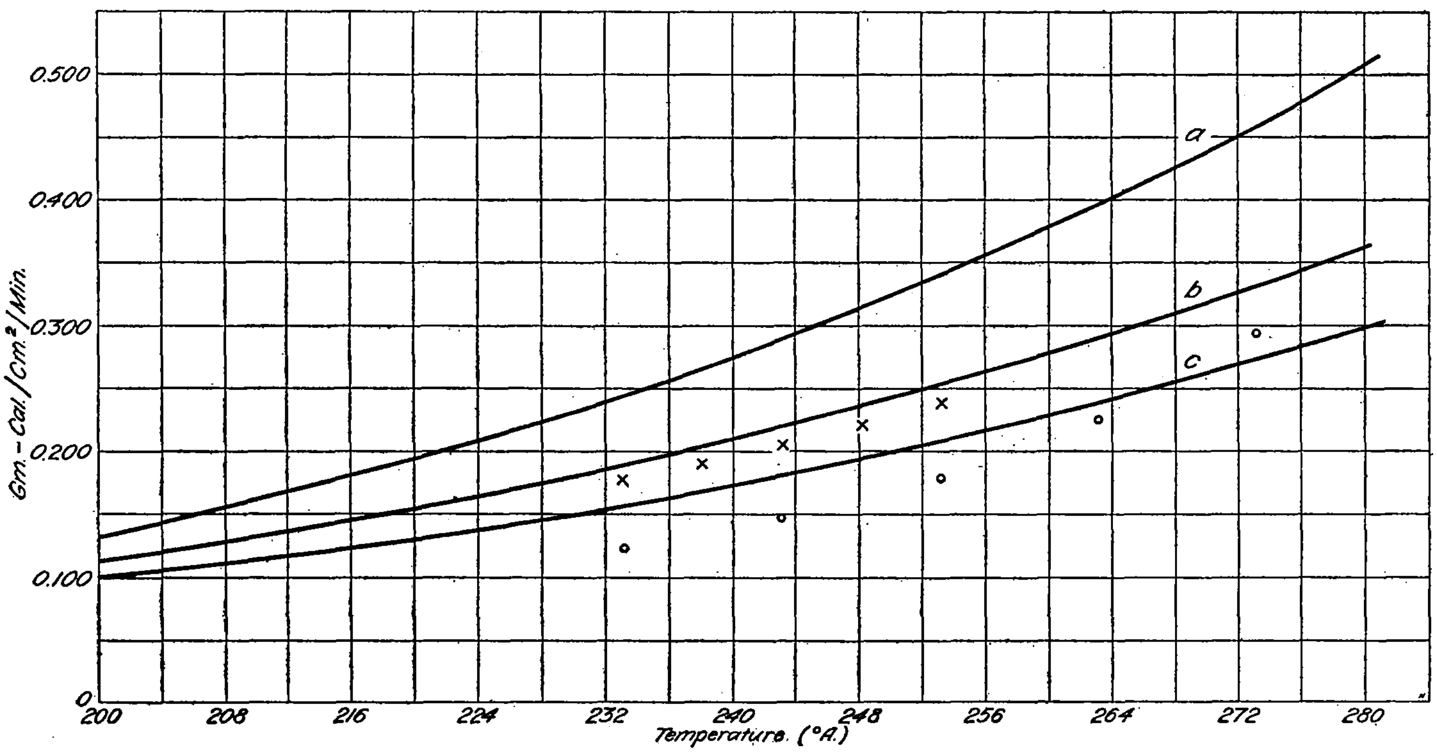

FIGURE 4.—(*a*) Radiation from snow surface; (*b*) radiation from a moist atmosphere of high CO_2 content; (*c*) radiation from a moist atmosphere of normal CO_2 content.

If the mean surface inversion were about 10° C. then the points would coincide almost exactly with curve (*c*). Also in figure 4, are plotted values of atmospheric radiation from Ångström's formula, with the constants given by Raman (**12**), $\frac{S}{\sigma T^4}=0.77-0.28\times10^{-0.055e}$, where S=atmospheric radiation, T=temp. of the instrument, σ= Stefan-Boltzmann constant, e=vapor pressure in mb.

From the above formula, the values of atmospheric radiation have been computed *under the assumption that the air is saturated.* At low temperatures, the points fall below the curves (*b*) and (*c*) and also those of the "Maud", but approach them with increasing temperature. The discrepancy at low temperatures should not be taken seriously, since the formula has not yet been verified by observations at those low temperatures. Indeed the "Maud" observations in figure 4, and those taken by Ångström at Abisko (68°21′ N, 18°47′ E.), indicate higher values of atmospheric radiation than found from the formula. This is explained by Mosby as being due to the

H_2O : 0.0001 gram, 0.003 gram, 0.046 gram, and 0.455 gram. The result was a series of approximately straight and parallel lines, which showed that the radiation could be represented by the equation $I=\text{const.}\times T^n$, where n=3.5.

The curves found from equations (*b*) and (*c*) are not shown in figure 4, since they cannot be distinguished from the actual curves.

If now we wish to find the mean temperature of a stratum of air, each of whose unit columns contains 0.15 millimeter precipitable H_2O and 0.03 gram CO_2, that is necessary to maintain a constant surface temperature of 253°, we move the 253° ordinate of curve (*a*) over to curve (*b*); the temperature at which this ordinate fits the latter curve is the desired air temperature. Likewise, the mean temperature of a stratum of air each of whose columns contains 1 millimeter precipitable water and normal CO_2 content, will be found to be 291°. In table 1 are presented air temperatures necessary for the maintenance of various equilibrium surface temperatures.

Table 1.—*Mean temperatures of a layer of moist air necessary to maintain given surface temperatures*

	From curve (b)		From curve (c)	
1	2	3	4	5
$t_{Surf.}$	t_{Air}	t_A-t_S	t_{Air}	t_A-t_S
0° C.	+28° (24)	28	+41° (42)	41
−10	+14 (12)	24	+29 (29)	39
−20	+1 (1)	21	+18 (17)	38
−30	−11 (−11)	19	+4 (4)	34
−40	−23 (−23)	17	−9 (−9)	31
−50	−35 (−35)	15	−21 (−21)	29
−60	−46 (−47)	14	−34 (−34)	26
−70	−59 (−58)	11	−47 (−47)	23

From the table, we see that as the surface temperature decreases, the magnitude of the inversion decreases. This is owing to the well-known shift of maximum intensity of radiation to larger wave-lengths as the temperature of the radiator decreases; as this happens, the ratio of energy in the transparent band of the water-vapor spectrum to the total amount of energy decreases, and the atmosphere radiates more nearly as a black body. This is shown in figure 4 by the convergence, with decreasing temperature, of the curves showing black body and atmospheric radiation. At very low temperatures, instead of inversions there would exist only isothermalcy.

In table 1, the temperatures in the first and the second columns are found to be almost linearly related, according to the equation

$$t_A = 1.18 t_S + 24 \text{ in °C.}, \ -70° \leq t_S \leq -20°,$$

or

$$T_A = 1.18 T_S - 25 \text{ in °A.}, \ 203° \leq T_S \leq 253°.$$

Likewise the temperatures in the first and the fourth columns conform very closely to the linear relation

$$t_A = 1.27 t_S + 42 \text{ in °C.}, \ -70° \leq t_S \leq 0°,$$

or

$$T_A = 1.27 T_S - 32 \text{ in °A.}, \ 203° \leq T_S \leq 273°.$$

The air temperatures computed from these formulae are included in table 1, enclosed by parentheses, and it is seen that the agreement with the true values is very close over most of the temperature range considered.

The magnitudes of the inversions shown in table 1 can be checked by comparing them with some large inversions that have been observed. The comparisons are shown in table 2.

Table 2.—*Magnitudes of observed inversions compared to those of computed inversions*

1	2	3	4	5	6	7	8
Station	Date and time	Temperature in the surface shelter	Height of top of inversion	Temperature of top of inversion as recorded on meteorograph	Temperature of top of inversion as observed by thermometer	Temperature of inversion top as given in column 2, table 1	Temperature of inversion top as given in column 4, table 1
Fairbanks, Alaska	Dec. 28, 1932:	°C.	m	°	°	°	°
	9.34 h	−40.5	1,160	−18.5	−17.0	−23.5	−9.0
	20.26 h	−41.0	840	−16.5	−18.0	24.5	−10.0
	Jan. 25, 1933:						
	9.48 h	−37.8	1,830	−19.8		−20.5	−6.0
Fargo, N. Dak.	Jan. 14, 1935:						
	4.29 h	−32.0	1,780	−12.7		−13.5	+1.5
Billings, Mont.	Jan. 14, 1935:						
	3.25 h	−23.2	800	−2.6		−3.0	+12.5

The temperatures in columns 7 and 8 of table 2 were found from table 1, using the temperatures in column 3 as the argument; and it is seen that the temperatures in column 7 are lower than the observed temperatures in column 5 (and 6), while those in column 8 are higher. In other words, by using Hettner's absorption data, for an atmosphere overcharged with CO_2, smaller inversions result than those found in Nature. On the other hand, using Weber and Randall's data for an atmosphere containing normal amounts of CO_2, the inversions are large enough to account for those observed. The inversions shown in table 2 would have been much larger if temperatures of the snow surface had been observed, since in all cases the surface temperature was observed in a shelter a meter or so above the snow surface, and it is well known that under the conditions here assumed the temperature of the snow is lower than that of the air a short distance above it. To illustrate this, in table 3 are presented some typical observations taken by Ångström at Abisko, Lapland (**14**), during a calm, cloudless polar night.

Table 3.—*Temperatures at different heights above a snow surface during a clear, calm night at Abisko, Lapland (after Ångström)*

Date	Time	Temperature at snow surface	Temperature 0.6 m above surface	Temperature 2.0 m above surface
	h.	°C	°C	°C
Jan. 12, 1916	11.00	−12.8	−8.4	−7.6
	12.00	−16.1	−10.6	−9.4
	13.30	−17.0	−12.0	−10.5
	16.00	−18.0	−14.1	−12.5

From table 3 we see that the temperature of the snow surface may be 5° C. lower than that of the air 0.6 meter above it; and 6° or 7° lower than that of the air 2.0 meters above. Hence, if the temperatures in column 3, table 2, were the true surface temperatures, the values shown in column 7 would be found to be several degrees lower and would disagree even more seriously than those in column 5. On the other hand, it is not likely that the temperatures in column 8 would be found to be lower than those observed, even if the true snow surface temperatures were used.

Returning to the question of radiative transfer between the atmosphere and the snow surface, we have seen that the surface temperature will remain unchanged at 253°A. provided that the temperature of the air remains unchanged. However, if we are dealing with the same column of air, it is evident from figures 3a, 3b that its temperature must decrease, since it is emitting more radiation than it receives. As mentioned above, the surface temperature will remain constant at 253° A. because it is receiving radiation from the atmosphere equal to the shaded areas under the 274° A. curve, figure 3a, and emitting an equal amount of energy which is represented by the total area under the 253° A. curve; but the atmosphere does not absorb all this energy that is being emitted by the snow-surface, because the *T*-radiation and about one-half the *S*-radiation escape to space, as indicated by the cross-hatched area under the 253° curve. Hence the mean temperature of any layer that contains 0.15 millimeter precipitable water and larger than normal CO_2 content must fall below 274°. With the initial lapse rate we have assumed, this will apply only to the lowest layers; and the lapse rate in the stratum including these levels will become practically isothermal. Since the temperature of the stratum is then lower than it was originally, it will send less radiation to the snow surface, and so the surface temperature will decrease. This process will continue to go on, the atmosphere losing energy to space mostly by way of the snow surface; and as the air temperature decreases, the original steep lapse rate will be transformed into approximate isothermalcy up to a certain height. This is illustrated in figure 5, where the original lapse rate is shown by a solid line and the new lapse rates by dashed lines. The lengths of unit columns containing 1 millimeter precipitable water are

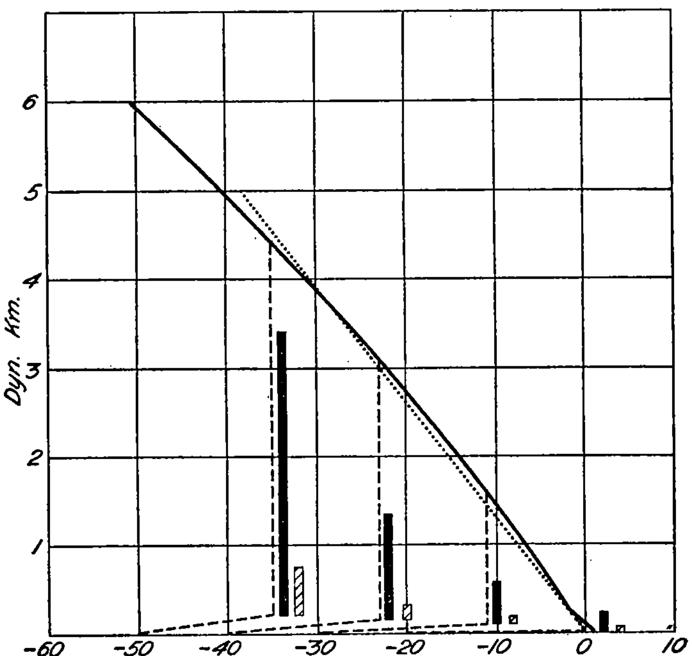

FIGURE 5.—Successive stages in cooling of polar maritime air over a snow surface during a calm, cloudless polar night.

indicated by shaded rectangles, and those containing 0.15 millimeter by cross-hatched rectangles.

A discussion of an example will show why the lapse rates will take on approximately the indicated successive shapes. Let us suppose that the temperature of the surface is −40° C. Then no layer of air, each of whose columns contains 0.15 millimeter precipitable water and 0.03 gram CO_2, can have a mean temperature higher than −23° C.; for if the mean temperature of a certain layer were higher, then the surface and underlying layers would receive more radiation than they would emit and consequently their temperatures would rise. Hence the surface temperature can fall to −40° provided only that the mean temperature of no layer above it is higher than −23°. Similarly it can be shown that by using the data of Weber and Randall, and assuming normal CO_2 content, the mean temperature of no layer of air can be higher than −9°. According to the ideal conditions assumed in this paper the inversion layer will be of infinitesimal thickness. However, in figure 5, for the sake of clarity, the inversions are represented by layers of finite thicknesses.

Looking back now, we see that since only a thin stratum of snow and air will initially be involved, the surface temperature will at first fall very rapidly to a certain value dependent on the maximum mean temperature of a layer of the atmosphere containing in one case 0.15 millimeter precipitable H_2O and 0.03 gram CO_2 and in the second case 1 millimeter precipitable H_2O and a normal amount of CO_2. As the surface temperature falls below this critical value, the temperature of a certain portion of the atmosphere must fall simultaneously; and its rate of fall will become slower as the temperature becomes lower, since for the same drop in temperature the surface must radiate to space a larger amount of energy, as seen from figure 5. Also, the rate of loss of energy to space by the surface becomes smaller with lower temperature; but a fuller account of the rate of decrease of temperature will be given later.

From the above discussion, we see that the top of the isothermal layer marks the height to which cooling from the surface has extended; hence, strictly speaking, this level should be considered to be the top of the polar continental air. The air above, characterized by a steep lapse rate, is, in this case, still polar maritime air. An exception to this general statement will be pointed out later. During January and February 1936, when polar continental outbreaks were very pronounced over the Middle West, the daily cross-sections now prepared at the Central Office of the Weather Bureau were analyzed according to the above principles, with satisfactory results.

From figure 5, it is seen that the lower the surface temperature, the smaller the inversion, and the higher the isothermal layer will extend; the top of the latter marks the height to which the influence of surface cooling has reached. To illustrate the phenomena by observations, in figure 6 are plotted points relating the height of the isothermal layer to its temperature at Fargo, N. Dak. (46°54′ N., 96°48′ W., elevation above m. s. l., 274 meters) during January and February 1936. In many of the soundings, ground inversions also existed, but some of the soundings showed steep lapse rates near the surface because of turbulence within the moving polar mass. Fargo is not situated in a true source region for polar continental air; and when polar air does reach it, the low surface temperatures characteristic of radiative

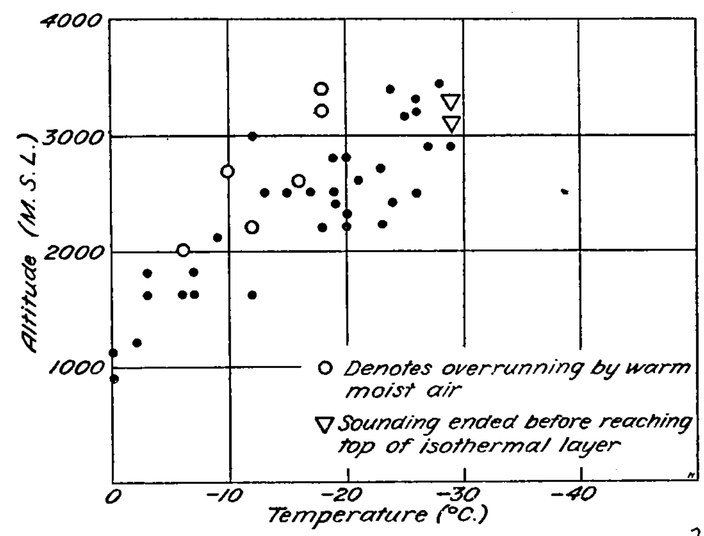

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FIGURE 6.—Temperatures and altitudes of isothermal layer above Fargo, N. Dak. (~~600~~ m. M. S. L.), January 1–February 19, 1936.

equilibrium are usually lacking because of the wind movement. Therefore no attempt has been made to correlate the magnitude of the ground inversion with the surface temperature.

Although the heights and temperatures of the "isothermal layer" are plotted in figure 6, it must be pointed out that not all of these layers were perfectly isothermal. A strong wind can destroy the lower portion of the isothermal layer, and after the wind dies down the air will cool in such a way that the lower portion of the isothermal layer becomes an inversion layer of considerable thickness. Also, as will be pointed out later, subsidence of the air mass will tend to transform the upper portion of the isothermal layer into an inversion layer. Hence in plotting figure 6, thick layers, above the surface inversion or adiabatic layer in which the temperature increase or decrease with elevation was less than 0.2°/100 meters, have been considered to have a uniform temperature equal to the mean temperature of the layer.

DIRECT RADIATION LOSS AND THE ROLE OF THE EMISSION LAYER

At this point it is desirable to examine to what extent the atmosphere can lose heat directly to space, and not by way of the surface. In figure 1, by a comparison of soundings (*a*), (*b*), (*c*), we see that because of the convergence of the curves with increasing height the main loss of energy must be by way of the surface, and will be mostly in the form of T-radiation. Nevertheless, an appreciable amount of energy is lost to space from the upper levels of the troposphere, and this loss has an important influence on the lapse rate in upper levels. From figure 1, sounding (*c*), we see that the isothermal layer has a temperature of about −41° C., and extends to 3,300 meters. However, from figure 5 we see that if the cooling is from below only, and the initial lapse rate of the air is that shown by the solid line, then an isothermal layer with temperature −41° C. should extend to about 5,000 meters. Now if in figure 1, sounding (*c*), we extend the isothermal layer to 5,000 meters and then draw an adiabatic lapse rate curve above, this latter curve will intersect the sounding at the next "significant" point. This curve is indicated as a dotted line extending from 3,300 meters to 6,200 meters, and would probably represent the actual lapse rate if cooling had occurred from below only. If this be true, then the area between this curve and the original sounding is nearly proportional to the energy lost to space directly from the atmosphere. That the troposphere at high levels does lose energy directly to space has been shown by Albrecht, (**16**), (**17**), (**18**), according to whom the rapid decrease with height of atmospheric water-vapor content, and consequently of absorbing power, results in a natural upper limit to the region of atmospheric absorption. This critical height, corresponding approximately to the height of the −50° C. isotherm in an atmosphere of normal moisture content, marks the top of the emission layer, about 3 kilometers thick, from which atmospheric radiation is lost to space. In Albrecht's example (**16**), the emission layer, as determined from Peppler's mean summer values of temperature for Germany and Süring's formula for specific humidity, extended from 9 kilometers (−41°) to 12 kilometers (−50°). Applying the analysis described above to sounding (*c*), we find independently that the emission layer in this case extends from 3,300 meters (−41°) to 6,200 meters (−52°), in good agreement with Albrecht's bounding temperatures. It is also interesting to note that in sounding (*c*) the tropopause is found immediately above the emission layer, showing that in this case the emission layer forms the natural limit of the troposphere, which Albrecht pointed out to be true for the temperate and polar regions.

Albrecht found from Hettner's absorption data [(**16**), p. 433, (**17**), p. 65] that the rate of loss of energy to space from the emission layer would be nearly equal to that of the selective radiation from water vapor and carbon dioxide at −50° C., or about 0.170 gram cal/cm²/min. As illustrated in figures 3a, 3b, the loss of energy to space from the surface takes place mostly through the transparent band; and, as can be seen in figure 4 by subtracting from the black body curve either of the two curves of atmospheric radiation, it is much less than that found by Albrecht for the emission layer. Now if Albrecht's value were correct, then it would clearly be impossible for a sunless, cloudless, motionless atmosphere, possessing initially a steep lapse rate, to cool more rapidly in lower than in higher levels. In other words the cooling would proceed in such a manner that a steep lapse rate would always exist. This is contrary to the lapse rates observed in polar continental air masses, which are small, as seen from the examples in figure 1 and from Willett's data for American polar continental air (**19**). Recently, Mügge (**35**) has presented curves showing rates of cooling for a tropical, subtropical, subpolar, and a polar atmosphere. The curves are based on earlier papers by Mügge and Möller (**36**), (**37**), and in each type of atmosphere, he finds ((**35**), p. 169) "* * * that in all latitudes the cooling is greater aloft than below, thereby continually creating atmospheric instability." This conclusion is hardly consistent with the temperature-height curve he presents showing the structure of polar air; according to this curve, the lower portion of the air is very stable, the temperature decrease between the surface and 4 kilometers being about 12° C.

Albrecht's high value of the loss of energy from the emission layer can probably be explained by the fact that he used Hettner's absorption data, which as we have seen seem too high as compared with those of other investigators. In a recent paper, Ramanathan (**20**) applied Hettner's absorption data in a first approximation to the solution of the problem of radiative equilibrium in the upper troposphere; however, instead of assuming a certain amount of absorption in the region 8–13μ, as Hettner observed, Ramanathan assumed complete transparency. Following Simpson's scheme, he assumed an initial lapse rate of 0.6, and used elementary layers of thickness 1 kilometer, each having a mean temperature differing from those of adjacent layers by 6° C. He found that an atmosphere having the assumed lapse rate was not in radiative equilibrium, and that the layers would tend to lose or gain energy at rates shown in table 4.

TABLE 4.—*Net radiation from different layers of the atmosphere*

[Unit=gm. cal/cm²/min.×10⁻⁴ (after Ramanathan)]

Atmospheric layer	Assumed mean temperature of layer	0.0015 cm. ppt. H_2O in stratosphere		0.01 cm. ppt. H_2O in stratosphere		0.03 cm. ppt. H_2O in stratosphere	
		100 percent rel. humidity in troposphere	25 percent rel. humidity	100 percent	25 percent	100 percent	25 percent
−3	205° A	−16	+8	+36	+24	+51	+24
−2	211	−70	−15	31	30	65	44
−1	217	−122	−40	17	18	60	39
1	223	−156	−93	−30	0	17	30
2	229	−158	−155	−77	−34	−39	5
3	235	−112	−197	−76	−78	−57	−39
4	241	−62	−166	−53	−91	−42	−65
5	247	−25	−121	−26	−90	−20	−52
6	253	−9	−84	−9	−61	−9	−51
7	260	-------	−35	-------	-------	-------	−32
Loss from the emission layer		−730	−896	−271	−354	−167	−239

In table 4 it is seen that the maximum loss of energy from the emission layer is about 0.090 gram cal/cm²/min, which is considerably less than that found by Albrecht;

and that the minimum is about 0.017 gram cal/cm²/min. As the moisture in the stratosphere increases, the emission layer becomes lower, and less energy is lost to space. However, an increase in the moisture in the troposphere causes the emission layer to become *higher*, and less energy is then lost to space. It would be valuable to extend Ramanathan's work beyond the first approximation, by the method used by Gowan (**21**), (**22**) to find the temperature distribution satisfying radiative equilibrium in the ozone region.

Now after the cooling process has gone on for some time, and the temperature of the lower atmosphere has become low, then the emission layer, which has been radiating energy to space continuously, can decrease in temperature, since it then receives less intense radiation from below and practically no radiation from above. Of course the very stable stratification below prevents any upward transport of heat by turbulence. As illustrated in sounding (*c*), figure 1, this will result in a certain upper limit to the height to which the isothermal layer can extend (probably about 3,500 meters), and the lapse rate above will be small. In these extreme cases of cooling, it would seem proper to call the whole troposphere polar continental air, since it is only after the lower portion of the atmosphere has cooled considerably by way of the surface that the emission layer can also cool. As the cooling continues, both from the surface and the emission layer, then the latter becomes lower, because its height, as shown by Albrecht, is a function mainly of temperature. Also, because of cooling, the lower atmosphere will contract. To estimate the magnitude of this latter effect, the heights of the 500 mb. surface were compared for soundings of type (*a*) and type (*c*), figure 1. Assuming the surface pressure to be the same, the difference was found to be about 500 meters. In figure 1, we see that the Ellendale sounding meets the tropopause at about 6.2 kilometers, while the As sounding meets it at about 8.1 kilometers; hence contraction of the lower atmosphere by cooling could not account for the difference in height of the tropopause at these two stations. Evidently other factors must enter, and we shall later examine the effects of radiation.

RATE OF DECREASE OF SURFACE AND FREE-AIR TEMPERATURES OVER THE CONTINENT

Let us now examine the rate of decrease of surface and air temperature, assuming that the loss of energy to space takes place from the snow surface. As stated above, the snow temperature will at first fall rapidly, from its initial value of 0° C., to −20° C. (or to −33°), since only a thin stratum of snow and air will cool. Below this temperature, the cooling will become slower and slower, since a larger and larger portion of the air column must be cooled.

Brunt [(*23*), p. 127] gives the following formula for the rate of decrease of surface temperature with time, for a calm, cloudless atmosphere resting above a homogeneous surface, where only conduction of heat from the underlying surface compensates for the loss of heat by nocturnal radiation:

$$\sqrt{t}=\frac{\sqrt{\pi}}{2}\frac{\rho c\sqrt{\kappa}}{E}(T_o-T),$$

where t=time in hours for surface temperature to fall from T_0 to T (from 273° to 253°, or to 240°); ρ=density of the surface=0.5 for old snow and 0.10 for new snow; c=specific heat of the surface=0.5 for snow; κ=conductivity of the surface=0.0011 for old snow and 0.00025 for new snow (**24**), E=effective nocturnal radiation (difference between black-body radiation at temperature T and return atmospheric radiation).

Now in computing the fall of temperature from 0° to −20° C. (or −33°), a mean value of E must be found over the temperature range considered; this can be done from curves (*a*), (*b*), and (*c*) in figure 4. The results are shown in table 5.

TABLE 5.—*Times for cooling of the snow surface from 0° to the critical temperature*

Absorption data	Critical temperature	Mean value of E in gm cal/cm²/min.	Time for cooling	
			Old snow surface	New snow surface
			Hours	*Hours*
Hettner (above normal CO_2 content)	−20° C.	0.11	1.5	0.02
Weber & Randall (normal CO_2 content)	−33°	.14	2.3	.03

In reality, due to the release of latent heat of condensation and the transport of heat downward from the air by mechanical turbulence, the time for cooling is larger than the values in table 5; but these may be regarded as minimum values.

In order for the surface temperature to fall below −20° C. (or −33° C.), the lower portion of the atmosphere must cool; and as shown above, the cooling will transform the original temperature-height curve into those of the forms shown by the dashed lines in figure 5. If the ordinate of this figure were pressure, then the amount of internal and potential energy lost after a certain interval of time would be proportional to the area between the original temperature-height curve and the new curve.

If T_i represents the initial temperature of some point at pressure p, in a unit air column, and T_f represents the final temperature of this point, the difference in energy (internal plus potential) of the air column is given by the following expression [(**25**), p.27]:

$$(1)\qquad U=U_i-U_f=-\frac{c_p.10^3}{g}\int_{po}^{p_h}(T_i-T_f)dp \text{ gm. cal.}$$

where p=pressure in mb. p_o, p_h=pressures at bottom and top of the air column, c_p=specific heat of dry air at constant pressure=0.240, g=acceleration due to gravity.

Since T_i represents the known initial temperature distribution and T_f the final distribution, which for the cooled portion of the atmosphere is a function of T_s, the temperature of the snow (as seen from table 1), therefore (1) may be written as a function of T_s:

$$(2)\qquad U=U(T_s).$$

As the saturated air cools, the water vapor will be condensed and the latent heat of condensation released. With an initial lapse rate of the type shown in figure 5, and assuming saturation, the amount of precipitable H_2O is about 1 gm.: and so when the column is cooled sufficiently, a considerable amount of heat may be released. The following equation has been found to represent with sufficient accuracy the latent heat released:

$$(3)\qquad U_w=3440-12.6T_A \text{ gm. cal.}$$

where T_A is the temperature in °A of the isothermal layer, which as we have seen is a linear function of T_s.

Due to the prevalence of subcooled water even at very low temperatures[3], only the latent heat of water has been taken into account, assuming that it equals 600 gm. cal. per gm. of condensed H_2O.

The presence of water droplets (or ice particles) in the atmosphere will have a certain influence on the radiative balance; but this is neglected under the assumption that the foreign particles are small and few in number, and fall rapidly to the surface in the absence of upward currents.

The energy (potential, internal, and latent) of the column will be lost to space principally through the T and S radiations from the snow surface; and the rate of loss will be

$$(4) \qquad E = I - R_A = \sigma T_s^4 - \kappa T_s^{3.5} \text{ gm. cal/cm}^2\text{/min.,}$$

where σ=Stefan-Boltzmann constant$=82.10^{-12}$, and $\kappa=980.10^{-12}$ or 800.10^{-12} according to whether curve (*b*) or (*c*) in figure 4 is taken to represent R_A.

Hence, the time required for cooling from $-20°$ C. (or $-33°$ C.) to T_s will be

$$(5) \qquad t = \int_{T_c}^{T_s} \frac{d[U(T_s) + U_w(T_s)]}{E(T_s)} \textit{ minutes,}$$

where in the one case $T_c=253^0$ and in the other, 240^0.

In order to evaluate (5), certain approproximations are introduced in order to obtain a simple expression for U: Instead of assuming an initial lapse rate that follows the condensation adiabat above a shallow adiabatic layer (as shown by the solid curve in figure 5), we assume that the initial lapse rate is linear and equal to 0.77 of the dry adiabatic rate (as shown by the dotted line in figure 5). After cooling has occurred the lower levels will become practically isothermal at temperature T_A. This isothermal layer is considered as extending downward to the surface, although actually there is a shallow inversion layer in which the temperature increases from T_s to T_A. The total energy of this layer is so small in comparison with that of the thicker layers considered here that it is regarded as negligible. Hence, in the simplified case, (1) becomes

$$(6) \qquad U = U_i - U_f = \frac{-C_p \times 10^3}{g} \int_{p_0}^{p_A} (T_i - T_A) dp;$$

for a linear lapse rate [redacted], 0.77 of the dry adiabatic, we have

$$(7) \qquad \frac{p}{1000} = \left(\frac{T_i}{273}\right)^{\frac{mg}{R \times 0.77}} = \left(\frac{T_i}{273}\right)^{4.5},$$

where $\frac{R}{m}$ is the gas constant for dry air, and g is the acceleration due to gravity.

Eliminating p from (6),

$$(8) \qquad U = \frac{-4.5 \cdot 10^6 \cdot C_p}{(273)^{4.5} \cdot g} \int_{273°}^{T_A} (T_i - T_A) T_i^{3.5} dT_i$$

$$= -66{,}860[-0.182(T_A/273)^{5.5} + T_A/273 - 0.818] \text{ gm. cal.}$$

For $T_A=249°$, U=1,042 gm. cal. as compared to about 1,092 gm. cal. determined from the original lapse rate of figure 5. Assuming a linear lapse rate of 0.77 the dry adiabatic thus results in an underestimate of U for the temperature range considered here. This difference is in part compensated by assuming that the loss of energy to space is only by way of the snow surface and not also by way of the emission layer; hence, the underestimate of U will be partially corrected by an underestimate in E.

Let us now compute the time required for the cooling of an atmosphere overcharged with CO_2, using Hettner's absorption data, remembering that in this case

$$(9) \qquad T_A = 1.18 T_S - 25.$$

Substituting (3), (4), (8) in (5), and eliminating T_A by means of (9), we find

$$(10) \qquad t = \int_{253}^{T_S} \frac{\frac{289}{(231)^{4.5}}[T_S - 21.2]^{4.5} - 304}{\sigma T_S^4 - \kappa T_S^{3.5}} dT_S \text{ minutes,}$$

where $\kappa = 980 \cdot 10^{-12}$ and $\sigma = 82 \cdot 10^{-12}$.

In order to integrate (10) it is necessary to expand $[T_S - 21.2]^{4.5}$; since the expansion converges very rapidly, only the first three terms need be retained, and hence (10) may be written as

$$(11) \qquad t = \frac{289}{(231)^{4.5}} \left\{ \int_{253}^{T_S} \frac{T_S^{4.5}}{\sigma T_S^4 - \kappa T_S^{3.5}} dT_S - 95.4 \int_{253}^{T_S} \frac{T_S^{3.5}}{\sigma T_S^4 - \kappa T_S^{3.5}} dT_S + 3540 \int_{253}^{T_S} \frac{T_S^{2.5}}{\sigma T_S^4 - \kappa T_S^{3.5}} dT_S \right\} - 304 \int_{253}^{T_S} \frac{dT_S}{\sigma T_S^4 - \kappa T_S^{3.5}}.$$

Each of the above integrals can be easily evaluated and the final expression for t becomes

$$(12) \qquad t = 81.2\{0.667y^3 + 11.95y^2 + 94.80y + 2606 \cdot \log_{10}(0.0837y - 1)\}$$
$$- 0.6204 \cdot 10^4 \{0.20x^5 + 0.021x^4 + 0.0023x^3 + 0.00029x^2 + 0.000041x\}$$
$$+ 595 \cdot 10^4 \cdot \log_{10}(0.0837 - x) - 1089 \cdot 10^4 \text{ minutes,}$$

where

$$y = T_S^{0.5}, \quad x = T_S^{-0.5}.$$

Similarly, it is possible to compute the time for the cooling of an atmosphere of normal CO_2 content, using Weber and Randall's absorption data.

In figure 7 are plotted the thermograms for the two cases studied. The inset figure shows the initial rate of cooling. In the first few hours the temperature falls very rapidly from 0° to the critical temperature, $-20°$ (or $-33°$), while the air temperature remains practically constant. The surface temperature can fall below $-20°$ (or $-33°$) provided only that the maximum air temperature also falls, so that at t=1.5 hours (or 2.3 hours), the temperature curves show discontinuities in slope; from this point on, the cooling rates become smaller and smaller, so that for the surface temperature to fall to $-40°$ requires about 15 days (or 2 days) of cooling, and to fall to $-60°$ requires about 62 days (or 23 days).

The large difference in the cooling rates for the two cases studied is due to the fact that the return atmospheric radiation is different in the two cases. A larger amount

[3] According to Frost (*26*) "at Fairbanks dense fog always accompanies temperatures of $-45°$ F. or lower. The records for January 1934, show 348 hours with $-45°$ F. or lower and during this time there were 320 hours of dense fog. On one occasion the fog prevailed for 155 consecutive hours." From 3 polar year airplane soundings taken through low temperature fog at Fairbanks it was found that their heights were 115 m., 200 m., and 75 m.

of return atmospheric radiation allows the surface temperature to fall a lesser number of degrees without a corresponding fall in the temperature of the free air; and because a larger amount of return atmospheric radiation implies a smaller amount of radiation passing to space, it will take more time for the surface temperature to fall to a given value. However, in the cooling of the free air, these two effects tend to compensate each other in the following manner: For the same temperature of the isothermal layer, the loss of radiation to space in one case takes place at a higher surface temperature than in the second case; this tends to equalize the rates of loss of energy, and so the free air thermograms do not diverge as much as the surface thermograms. For the isothermal layer to cool to −10° takes about 3 days as against 2 days; to −20°, 11 days as against 7 days; and to −30°, 25 days as compared to 18 days.

Alaska Section", published by the Weather Bureau, the writer obtained the minimum temperatures that occurred during each of the above months, except the value for January 1934, which was taken from Frost (**26**). The number of clear days also was noted, and to this was added one-half the number of partly cloudy days to find the percentage of clear sky during the month. It was not until the night of October 19–20 that the temperature fell below −20° C., so that this date can be taken as the zero-point on the time scale. In other words, from this date on, the temperature of the free-air began to decrease by surface cooling in the manner described above. From the thermograms in figure 7 were found the number of days necessary for the temperatures to decrease to the observed values. The results, together with the monthly totals of unmelted snow as published in the "Climatological Data", are shown in table 6.

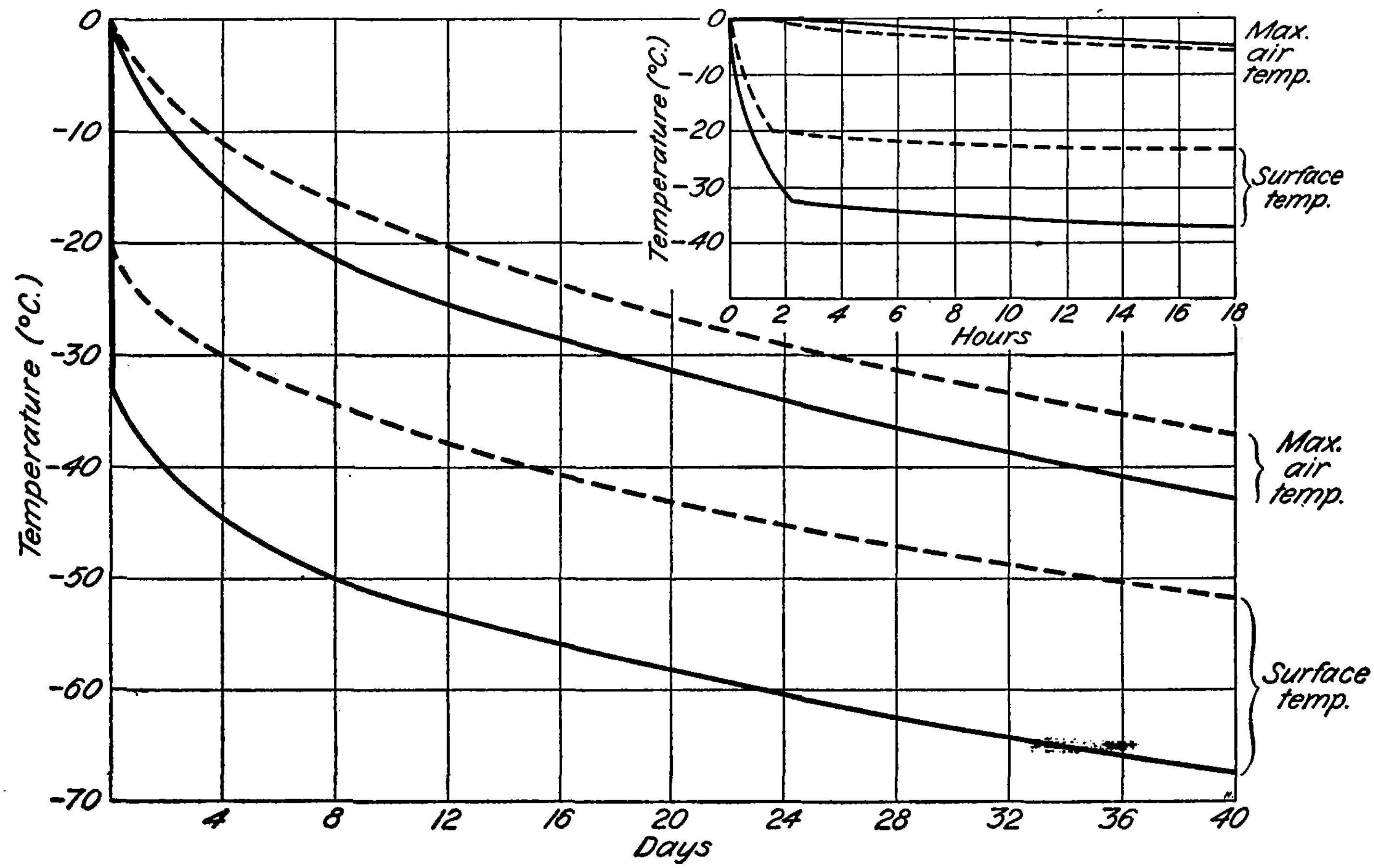

FIGURE 7.—Computed thermograms based on (*a*) Hettner's data in atmosphere of abnormal CO_2 content (shown by broken lines; (*b*) Weber and Randall's data in atmosphere of normal CO_2 content (shown by solid lines).

The rates of cooling presented in figure 7 must be regarded as *maximum rates of cooling*, since *absolutely calm, cloudless conditions have been assumed*. Clouds, or even a small wind movement, would delay the cooling considerably. In order to decide which of the two sets of thermograms is closer to reality, it would be necessary to study the thermogram of a station at a high latitude and situated on a snow-covered continent (or thick ice cover), and to make allowances for cloudiness, wind movement, and advection of colder or warmer air. As this is impossible, we must be satisfied with a less rigorous check. This was done by examining the march of minimum temperature at Fort Yukon, Alaska (66°34′ N, 145°18′ W, 140 meters above mean sea level), from August 1933 to January 1934. From the "Climatological Data for the

TABLE 6.—*March of monthly minimum temperatures at Fort Yukon, Alaska (66°34′ N, 145°18′ W, 139 meters above m. s. l., thermometer shelter 1.3 meters above the ground)*

1	2	3	4	5	6	7	8
Date	Inches of unmelted snow for the month	Minimum temperature	Number of days from Oct. 20	Per cent clear sky	Number of clear days	Number of days—Hettner—high CO_2 content	Number of days—Weber & Randall—normal CO_2 content
1933–34		° *C.*					
Aug. 28	0	0					
Sept. 2	8	−8					
Oct. 20	7	−31	0	42	0	0	0
Nov. 27	13	−42	38	40	16	18	3
Dec. 31	2	−54	72	68	38	43	13
Jan. 14		−61	86	78	49	66	25

In table 6, we see that the number of clear days which elapsed from October 20 to the dates of the various monthly minimum temperatures as given in column 6, is below that of column 7, and above that of column 8, where it must be remembered that the latter values are in terms of "ideal" days—that is, full days during which calm, cloudless, sunless conditions prevailed. Now, the number of days given in column 6, if reduced to "ideal" days, would be much less, because some sunlight, cloudiness, and wind movement undoubtedly occurred during the period. Also it is possible that a temperature 1° higher than the minimum may have occurred at some earlier day in the month, and this too would reduce the number of days in column 6. Even so, the values in column 7 are much higher than those in column 6, showing that the combination of Hettner's absorption data and an abnormal CO_2 content leads to a rate of cooling much slower than that observed in Nature. On the other hand, using the values of Weber and Randall, together with a normal CO_2 content in the atmosphere, a rate of cooling is found which is large enough to account for the march of minimum temperature observed at Fort Yukon.

It may be argued that the low temperatures observed at this station were not created in the vicinity by radiational cooling but rather are due to the importation of colder air from other localities. This may be true, and the fact that −54° was observed at this station on December 31 may mean that it was observed at an earlier date at another, colder locality. This, however, would still mean that the values in column 7 are too high, unless a serious error were made in the selection of October 20 as the zero-point on the time scale. For example, it might have happened that some other colder station reported a temperature below −20° before October 20 and several extra days of cooling occurred before this air moved into Fort Yukon. This would increase the number of days in column 6. To check this possibility, the minimum temperatures for all the weather stations in the Yukon Valley, as published in the October "Climatological Data," have been inspected. On October 17, Eagle (about 200 miles southeast of Fort Yukon) was the first to report a minimum temperature below −20° C. (−21°). On October 19 several more stations reported minimum temperatures below −20°, and finally on October 20 Fort Yukon reported the same. Now if we should start counting the days from October 17, this would increase the number of clear days in column 6 by at most 3. Even if this were done the values in column 7 would still be too large.

Importation of cold air from north of the Yukon Valley is hindered by the Endicott Range; and this air, even if brought down from the north, would usually be warmer than the interior air, since it is well known that air over the frozen ocean is in general warmer than that over a continental snow cover [(**27**) p. 96 et seq.]. This is illustrated by comparing the mean December temperature for Point Barrow, Alaska (71°23′ N., 156°17′ W., on the Arctic coast) and that for Fort Yukon. The normal temperatures for December are −25.2° C. and −29.6°, respectively; while the mean December 1934 temperatures were −25.8° and −37.3°, respectively.

Another possibility of error lies in the initial lapse-rate assumed. It might be argued that during the summer the air over the Arctic Ocean is much cooler than assumed in figure 5, and that less time would therefore be required to cool the air to the values observed at Fort Yukon. This would then decrease the values in column 7, table 6, and make them agree better with the observed rates of cooling. However, it is a well-known fact that the sea-level temperature over the Arctic during the summer is about 0° C. [(**27**) p. 89]; and assuming a lapse-rate of the kind shown in figure 5, that is, air in convective equilibrium (above a shallow adiabatic layer) with an ocean surface of temperature 0° C., means that this air would represent the coldest type of air possible over the Arctic during the summer. For this reason, and because of the assumptions of a calm, cloudless, sunless atmosphere, the rates of cooling found above must be regarded as *maximum values*. It must be realized that this is true only for the northern hemisphere, since over the Antarctic the summer temperature is much lower than 0° C.; the mean December ice barrier temperature observed by Simpson was about −6° C. [(**28**) p. 31, et seq.] and the mean Antarctic Plateau temperature much lower.

HEIGHT AND TEMPERATURE OF TROPOPAUSE OVER POLAR MARITIME AND POLAR CONTINENTAL AIR

In figure 8 we have reproduced the Ås soundings from Palmén's paper [(**1**) fig. 5]. The time interval between

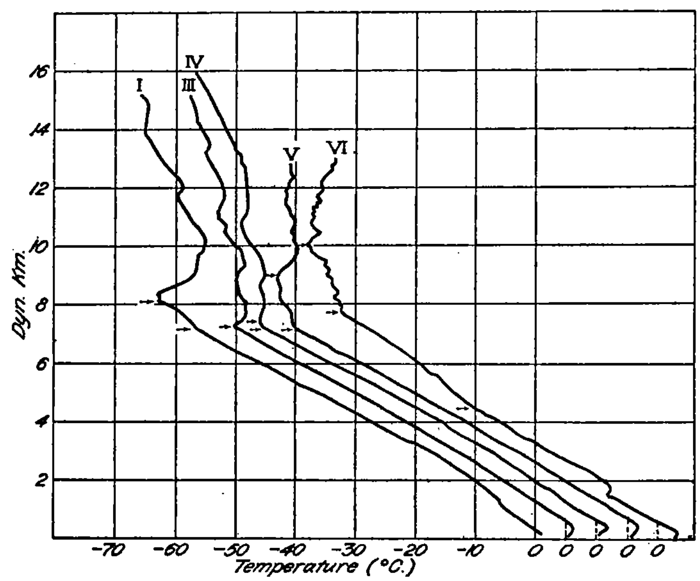

FIGURE 8.—Soundings at Ås. Arrows designate the tropopauses and noteworthy significant points (after Palmén).

soundings I and III is 8 hours; we notice that the tropopause in III is at about 7.3 kilometer, or about 0.8 kilometer lower than in I, and its temperature is −55.4° C. as compared to −62.8° C. in I; but, as Palmén pointed out, in sounding I a discontinuity in lapse rate already appears at 7.2 kilometers at a temperature of −56.6° C. Palmén suggests that this point is at the cold front which had passed Ås during the preceding night; but it appears more reasonable to interpret it as the first sign of a new tropopause, since the tropopauses found in soundings III, IV, V are at approximately the same height and have approximately the same temperature as this discontinuity. (As Palmén points out, the first signs of a second, higher tropopause found in soundings V and VI, apparently are due to an approaching disturbance west of Iceland.) The destruction of the original tropopause in I can be explained by its being bounded above and below by strata of warmer air; the temperature of the top of the tropopause inversion is −54.8°, which is not far different from the temperature of −56.6° found at the discontinuity below the tropopause, while the temperature at the bottom of the inversion is −62.8°. From considerations of radiative balance, it appears that the temperature of the latter point must rise until it becomes equal to that of the bounding air, provided the inversion is not maintained by convection. An exception would occur in the presence of a discontinuity in moisture, which would allow the sharp inversion to be maintained in the manner of a "dry inversion" (**29**). However, at the low temperatures considered here, the amount of water vapor is so small that differences in it can hardly influence the radiative balance. Hence, provided convection is absent, the original tropopause in sounding I must vanish and the lower discontinuity become the new tropopause. In table 7 (reproduced from Palmén's paper) are presented the heights, temperatures, and potential temperatures at high-level significant points on the Ås soundings:

TABLE 7.—*High-level significant points in the Ås soundings*

[After Palmén]

Soundings 1933	Ht. in dyn. M.		Temperature in ° C.		Pot. temperature	
	H_1	H_2	t_1	t_2	θ_1	θ_2
No. I Feb. 2, 15^{11}	7,196	8,146	−56.6	−62.8	292.8	297.2
No. III Feb. 2, 23^{04}	7,265	----------	−55.4	----------	294.8	----------
No. IV Feb. 3, 3^{00}	7,141	7,401	−55.0	−56.0	293.2	295.6
No. V Feb. 3, 6^{55}	7,159	8,912	−55.3	−58.4	292.2	312.7
No. VI Feb. 3, 13^{10}	7,758	10,000	−52.6	−58.0	302.4	327.2

Each of the soundings except III has two significant points; and as pointed out above, the last two soundings show the influence of a rising tropopause apparently due to an approaching disturbance from the west. Palmén mentions that the potential temperatures of H_1 are almost constant from soundings I to V, but from the table we see that the *actual temperatures* are more nearly constant. This remarkable agreement of temperatures along the tropopause, evident during a period of more than 16 hours, indicates that the −55° or −56° isotherm marks the top of the emission layer over Ås. As Albrecht pointed out, the emission layer forms the natural top of the tropopause in high and middle latitudes, and from the mean summer values over Germany he found that the temperature of the top of the emission layer was about −50°; in his second paper (**17**), from a theoretical consideration, he found −53°; in the Ellendale sounding (fig. 1), from an independent analysis, the writer computed a temperature of −52°. Although the mean temperature of H_1 from the Ås soundings (−55.6°) is slightly lower than the above values, it appears that this isotherm also marks the top of the emission layer over Ås.

As pointed out by Albrecht and others (**17**, **30**), the tropical tropopause is higher than the emission layer, because of the continuous convection extending to high levels. Radiation currents both from the warmer emission layer and the warmer stratosphere tend to destroy the high cold tropopause, but convection dominates and maintains it. Now, when polar continental air leaves its source region and crosses an open ocean surface, then violent convection ensues; and although it is not yet possible to compute the height of this convection, it is probable that, initially, when the air temperatures are very low it extends to about 8 or 9 kilometers, a few kilometers above the emission layer. However, when the air (now polar maritime) passes over land, the tropopause is no longer maintained at its original height by convection, and its height must now be determined by the emission layer. Radiative balance will wipe out the original tropopause; and a lower, warmer one will form just above the emission layer. As the air moves over a snow surface and cooling of the lower atmosphere occurs, then, as pointed out above, the lower tropopause can contract about 500 meters. This will tend further to lower the tropopause and to raise its temperature. In table 7 we see that the new mean height and mean temperature of the new tropopause (H_1) are about 7.2 kilometers and −56°, respectively. In sounding (*c*), figure 1, the height and the temperature of the tropopause at Ellendale are about 6.2 kilometers and −52°. Hence, we see that after the new tropopause forms above the emission layer, the differences in height and temperature of the tropopause can be very nearly explained by its lowering and adiabatic warming due to the cooling and contraction of the troposphere.

From the arguments presented above we should expect a winter polar continental tropopause to be lower and warmer than a polar maritime tropopause. To verify this by observations, in table 8 is presented a portion of Palmén's table [(**31**), table 1], that gives the mean temperatures of polar (maritime) air over England for the winter half-year (October-March, 1924–29), together with polar continental data for North America. Palmén selected each of the soundings after a careful analysis of the synoptic situation so that they are representative of individual air masses, and grouped them according to high and low surface pressure. Although Palmén calls the air "polar", we assume on account of the maritime exposure of England that it is polar maritime. No other comparable group of observations at high latitudes, giving the mean winter structure of polar continental air to high levels, seems to be available. The observations made at Pawlowsk, Russia (59°41′ N, 30°26′ E), 1902–09, are available only as monthly means, and cannot be used since they probably include many cases of modified polar maritime and tropical air. However, in table 8 are shown means of the sounding balloon observations taken at Ellendale, N. Dak., in the winter half-year of 1932–33, during pronounced polar continental outbreaks. These observations have been discussed by Ballard (**32**); they were taken in two series a month, each series comprising three soundings made within 18 hours, the first at local noon, the second at midnight, and the third at 7 the next morning. Ballard attempted to correct the daytime observations for heating of the bimetal due to insolation and these corrections are included in table 8; for the noon flights they range from −1° C. at the surface to −4° at 10 kilometers, and for the morning flights from 0° at the surface to −1° at 10 kilometers.[4] Since some series did not include all three soundings, means of each series were computed and these were used to find the winter means, equal weight being given to each series. Five series were taken during pronounced cases of polar continental outbreaks in the winter half-year on the following dates: December 14–15, 1932; December 29, 1932; January 11–12, 1933; February 8–9, 1933; March 8–9, 1933. The individual soundings may be seen in Ballard's paper.

[4] These corrections are not included in the soundings reproduced in figures 1 and 9, since they were found from studying the mean differences and should therefore be applied only to mean values.

TABLE 8.—*Thermal Structure of Polar Maritime and Polar Continental Air*

Height in kilometers	England				Ellendale, N. Dak.	
	1022 mb.		996 mb.		1017–1030 mb.	Number of soundings
	Temperature	Number of cases	Temperature	Number of cases	Temperature	
	° C.		° C.		° C.	
Surface	+4.4	31	+6.7	21	−18.1	12
1	−1.1	31	−0.9	21	−20.9	12
2	−5.1	31	−6.7	21	−21.7	12
3	−9.8	31	−13.9	21	−25.0	12
4	−16.1	31	−19.6	21	−30.5	12
5	−22.8	31	−27.1	21	−35.7	12
6	−29.5	31	−34.6	21	−42.1	12
7	−36.6	31	−41.4	21	−46.7	12
8	−43.2	31	−46.2	20	−50.1	12
9	−49.7	31	−48.7	20	−51.5	12
10	−54.3	31	−49.9	20	−51.4	12
11	−57.0	31	−49.7	20	−50.8	12
12	−56.5	31	−49.7	18	−51.6	12
13	−54.4	29	−50.5	17	−51.1	12
14	−55.2	26	−51.4	14	−52.3	11
15	−55.9	21	−52.1	12	−53.7	9
16	−56.1	21	−53.3	12	−52.9	8
TROPOPAUSE						
H km.	10.70		8.55		7.4	
T	−58.8		−50.7		−50.3	

In table 8, by comparing polar maritime air in the neighborhoods of low pressure and of high pressure, we see that the former has a colder troposphere (except at the surface layer), a lower, warmer tropopause, and a warmer stratosphere. On the other hand, polar continental air has a much colder, stable troposphere than either of the two types of polar maritime air, and a lower tropopause whose temperature is slightly higher than that of the low-pressure type of maritime air but much higher than that of the high-pressure type; however, the actual stratosphere temperatures for the continental air seem to be between those of the two types of maritime air, although closer to those of the low-pressure type.

An explanation of these differences will not be attempted here, but the following important points should be emphasized. The lowest tropopause of the three types of air is found at Ellendale, a station about 600 kilometers south of the latitude of England, during periods of higher than normal surface pressure (the following are the respective mean sea-level pressures for each series: 1,030 mb., 1,017, 1,021, 1,029, 1,024). The temperature of this tropopause is even slightly higher than that of the warmest type of winter tropopause which occurs in England (in the neighborhood of a deep occluded cyclone).[5] The low, warm tropopause found over Ellendale during polar continental outbreaks probably could be found over all deep polar continental air masses; and its evolution from a higher, colder tropopause over polar maritime air is probably explained by the reasoning above, viz, the cessation of convection as the polar maritime air leaves the open ocean, thus allowing the emission layer to determine the new tropopause, and the contraction of the troposphere due to cooling.

THE EFFECTS OF SUBSIDENCE

As pointed out by Namias (**33**), surfaces of subsidence in American polar continental air extend over large areas and are also surfaces of constant potential temperature. Subsidence inversions will naturally form where some small inversion or weakening of the lapse rate already exists. Formerly it has been thought that radiative transfer across a moisture discontinuity or haze line would cause the initial weakening of the lapse rate, and subsidence would accentuate this into an inversion (**34, 33**); but from the above, it is seen that there also exists another explanation of the initial discontinuity in lapse rate that seems more plausible in the case of subsidence inversions covering large areas. If a homogeneous polar maritime air mass moves over a broad horizontal snow surface, then, after a certain interval of time, the isothermal layer

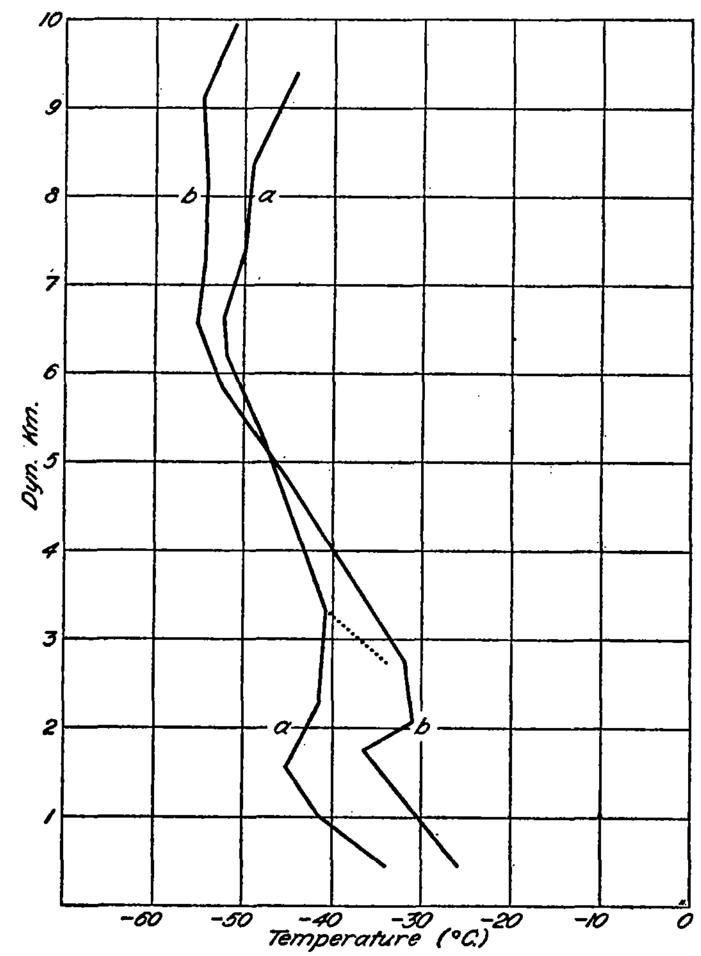

FIGURE 9.—Soundings at Ellendale, N. Dak., (*a*) February 8, 1933, 13.28 h.; (*b*) February 9, 1933, 7.42 h., local time.

will extend up to a certain height which will be nearly the same over the entire area, and so the top of the isothermal layer will have practically the same potential temperature. When the air mass moves southward and subsidence begins, the top of this layer will naturally tend to become the top of the subsidence inversion and will be characterized by almost constant potential temperature over its entire surface. To give an example of this phenomenon, portions of two balloon soundings made at Ellendale, N. Dak., on February 8–9, 1933, have been plotted on figure 9. The first of these soundings [curve (*a*), also plotted in fig. 1] was taken at 13.28 h. on the 8th; the second [curve (*b*)], made 18 hours afterward, shows a higher temperature from the surface to about 5,000 meters, the maximum difference being 12° C. at about 2,000 meters, although the surface temperature increased

[5] The temperature differences may be even larger than those shown in table 8, since it is unlikely that the English daylight soundings were corrected for insolation, as were the Ellendale soundings.

by only 8°. Evidently subsidence occurred above the well-stirred surface layer and this is verified by the close agreement of the potential temperatures at the top of the stable layers, 286° for sounding (*a*) and 288° for sounding (*b*). The decrease in thickness of the stable layer must be compensated by horizontal divergence, as Namias brought out in his example. According to his treatment of three well-defined cases of subsidence, the lapse rate of the air above the inversion decreased as the inversion became lower, but the decrease was much larger than that computed for a layer sinking without any change in its horizontal cross-section. Hence, horizontal divergence was necessary to explain the increased stability, since this process would bring together isotherms of potential temperature. One of Namias' examples showed a vertical contraction of 50 percent as the inversion subsided from 3,200 meters to 2,200 meters. With this in mind, Ballard's objection to subsidence in the soundings treated in figure 9 can carry little weight. In comparing the two soundings, Ballard (**32**) found that the mass of air from the surface to 5,000 meters decreased from the first sounding to the second, mostly because of increased temperature. This seemed to indicate that subsidence could not have occurred, since if it had, the mass of air from the surface to 5 kilometers would have had to increase. However, there is no difficulty in explaining the loss of mass if we assume that horizontal divergence occurs during subsidence.

The warm, dry air above the subsidence inversion will in most cases be the subsiding polar maritime air, since the subsidence inversion will generally tend to form at the top of the isothermal layer, which really marks the top of the polar continental air. Hence, many of the so-called "subsidence inversions" are really "fronts" which, however, may be accentuated by subsidence.

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